ON NEUTRAL OPERATIONS OF \((n,m)\)-GROUPS

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ABSTRACT. In this paper proposition on \(\{1,n-m+1\}\)-neutral operations of \((n,m)\)-groups is proved.

1. Preliminaries

Definition 1.1 ([1]). Let \((Q;A)\) be an \((n,m)\)-groupoid \([A:Q^n \to Q^m]\) and let \(n \geq m + 1\) \([n,m \in N]\). Then:

(a) we say that \((Q;A)\) is an \((n,m)\)-semigroup iff for every \(i, j \in \{1,\ldots,n-m+1\}, i < j\), the following law holds
\[
A(x_{i-1}^i, A(x_{i+n-1}^i, x_{n-1}^{2m-m})) = A(x_{j-1}^j, A(x_{j+n-1}^j, x_{n-1}^{2m-m}))
\]
\([\forall i,j - \text{associative law}]\); and

(b) we say that \((Q;A)\) is an \((n,m)\)-group iff \((Q;A)\) is an \((n,m)\)-semigroup and for every \(a_{n}^{m} \in Q\) there is exactly one sequence \(x_{1}^{m}\) over \(Q\) and exactly one sequence \(y_{1}^{m}\) over \(Q\) such that the following equalities hold
\[
A(a_{n-m}^{m}, x_{1}^{m}) = a_{n-1}^{m+1},
\]
\[
A(y_{1}^{m}, a_{n-m}^{m}) = a_{n-1}^{m+1}.
\]

Remark 1.1. A notion of an \((n,m)\)-group was introduced by Ć. Čupona in [1] as a generalization of a group \((n-\text{group, cf. [5]}\). The paper [2] is mainly a survey on the known results for vector valued groupoids, semigroups and groups (up to 1988).

Definition 1.2 ([3]). Let \((Q;A)\) be an \((n,m)\)-groupoid and \(n \geq 2m\). Let also \(e\) be a mapping of the set \(Q^{n-2m}\) into the set \(Q^{m}\). Then, we say that \(e\) is a \(\{1,n-m+1\}\)-neutral operation of the \((n,m)\)-groupoid \((Q;A)\) iff for all \(x_{1}^{m} \in Q^{m}\) and for every sequence \(a_{1}^{n-2m}\) over \(Q\) the following equalities hold
\[
A(x_{1}^{m}, a_{1}^{n-2m}, e(a_{1}^{n-2m})) = x_{1}^{m},
\]
and
\[
A(e(a_{1}^{n-2m}), a_{1}^{n-2m}, x_{1}^{m}) = x_{1}^{m}.
\]

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For $m = 1$, $e$ is an $\{1, n\}$–neutral operation of the $n$–groupoid $(Q; A)$. Cf. [5].

See, also [4].

**Proposition 1.1** ([3]). Every $(n, m)$–groupoid $(n \geq 2m)$ has at most one $\{1, n - m + 1\}$–neutral operation.

See, also [4].

**Proposition 1.2** ([3]). Every $(n, m)$–group $(n \geq 2m)$ has an $\{1, n - m + 1\}$–neutral operation.

See, also [4].

2. Results

**Theorem 2.1.** Let $(Q; A)$ be an $(n, m)$–group, $e$ its $\{1, n - m + 1\}$–neutral operation (cf. 1.5) and $n > 2m$. Then, for every $a_i^{n-2m}, \ x_1^m \in Q$ and for all $i \in \{1, \ldots, n - 2m + 1\}$, the following equalities hold

\[
A(x_1^m, a_i^{n-2m}, e(a_i^{n-2m}), a_i^{-1}) = x_1^m
\]

and

\[
A(a_i^{n-2m}, e(a_i^{n-2m}), a_i^{-1}, x_1^m) = x_1^m.
\]

**Proof.** Let

\[
F(x_1^m, b_1^{n-2m}) \overset{\text{def}}{=} A(x_1^m, b_1^{n-2m}, e(b_1^{n-2m}), b_1^{-1})
\]

for all $x_1^m, b_1^{n-2m} \in Q$.

Whence, we obtain

\[
A(F(x_1^m, b_1^{n-2m}), b_i^{n-2m}, e(b_i^{n-2m}), b_i^{-1}) =
\]

\[
= A(A(x_1^m, b_i^{n-2m}, e(b_i^{n-2m}), b_i^{-1}), b_i^{n-2m}, e(b_i^{n-2m}), b_i^{-1})
\]

for all $x_1^m, b_i^{n-2m} \in Q$. Hence, by definition 1.1 and by definition 1.3, we have

\[
A(F(x_1^m, b_1^{n-2m}), b_i^{n-2m}, e(b_1^{n-2m}), b_i^{-1}) =
\]

\[
= A(x_1^m, b_i^{n-2m}, A(e(b_1^{n-2m}), b_i^{-1}, b_i^{n-2m}, e(b_1^{n-2m})), b_i^{-1}),
\]

i.e.

\[
A(F(x_1^m, b_1^{n-2m}), b_i^{n-2m}, e(b_1^{n-2m}), b_i^{-1}) =
\]

\[
= A(x_1^m, b_i^{n-2m}, e(b_1^{n-2m}), b_i^{-1})
\]

for every $x_1^m, b_1^{n-2m} \in Q$.

In addition, hence, by definition 1.1 (cancelation), we obtain

\[
F(x_1^m, b_1^{n-2m}) = x_1^m
\]

for all $x_1^m, b_1^{n-2m} \in Q$, whence we have (1).

Similarly, we obtain, also, (2).

**Remark 2.1.** For $m = 1$ $(Q; A)$ is an $n$–group. See, also proposition 1.1–IV in [5].
REFERENCES


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